Hypersonic Characteristics of Sharp Cones at Angles of Attack

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Theme

THIS paper presents an analytic method of computing pressure coefficients and aerodynamic characteristics of sharp, slender, right-circular cones moving at hypersonic velocities in a rarefied atmosphere (Newtonian flow regime) which more accurately predicts experimental results than do other published methods, particularly when the angle of attack exceeds twice the cone half angle. The novel feature of this "Extended Newtonian Theory," relative to the existing "Modified Newtonian Theory" of Lees, is its incorporation of the small but nonzero surface pressure distribution downstream of the flow separation point. Aerodynamic parameters predicted by the two theories are compared with experimental values.

Content

For the hypersonic speeds and high altitudes where Newtonian impact theory is assumed to apply, the force on the body surface is obtained by computing the change in momentum of the particles of air that impinge on the surface of the body; frictional forces caused by the flow of air tangential to the body are neglected. Under these conditions the aerodynamic forces on the body and the various aerodynamic parameters can be computed when the pressure distribution on the body surface is known.

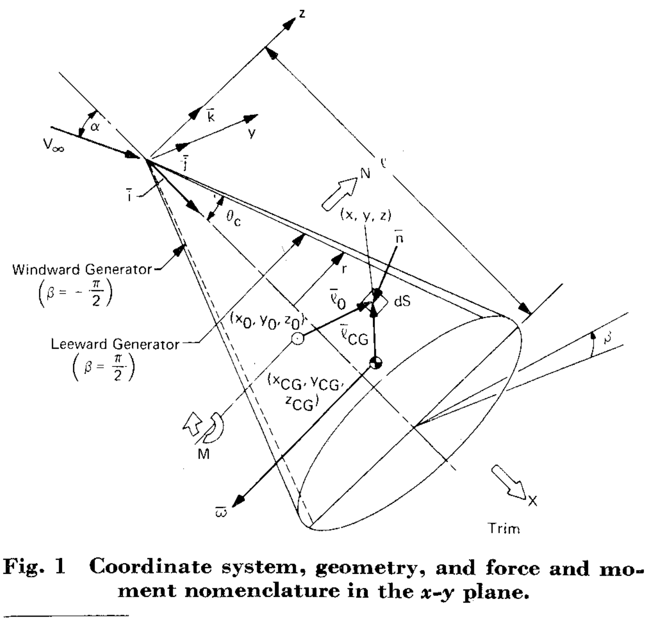

Fig. 1 Coordinate system, geometry, and force and moment nomenclature in the x-y plane.

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The pressure p acting on any element of the body surface is usually expressed as a pressure coefficient, $C_p = p/q$, where q is the dynamic pressure of the free stream relative to the body, $\rho V_\infty^2/2$, ρ is the air density, and V_∞ is the flight velocity.

In our analyses we express the pressure coefficient on the cone-surface in terms of its value $C_{p_{WG}}$ on the windward generator of the sharp cone (i.e., the line on the cone surface from tip to base that is in effect the leading edge of the cone; see Fig. 1 at $\beta = -\pi/2$), and D, the ratio of the C_p at other angular positions β around the cone to $C_{p_{WG}}$. Thus, at any point on the cone, $C_p = C_{p_{WG}}D$. According to Modified Newtonian Theory, for steady-state conditions

$$C_{p_{WG}} = K \sin^2(\alpha + \theta_c) \quad (1)$$

where $K = 2$ for Newtonian Impact Theory, or $K = C_{p_{STAG}}$ for Modified Newtonian Theory; α = angle of attack; and θ_c = cone half-angle.

In the Modified Newtonian Theory the maximum value of $C_{p_{WG}}$ occurs when the cone is at an α at which the forward generator is normal to the flow, where $\alpha + \theta_c = \pi/2$, and $C_{p_{WG}} = C_{p_{STAG}}$. However, at very high α's, as pointed out by Amick,[1] the flow must pass through an oblique shock and change direction. Hence the direction of the velocity of the impacting particles would not be normal to the surface of the cone at the windward generator when $\alpha + \theta_c = \pi/2$, and there would be some other value of $\alpha + \theta_c$ at which $C_{p_{WG}}$ would be a maximum.

The resultant analysis requires that the Newtonian prediction for the $C_{p_{WG}}$ incorporate two modifying factors:

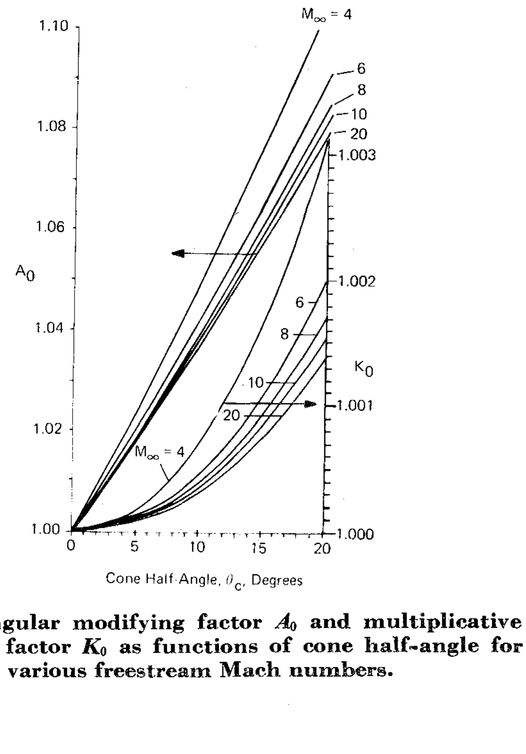

Fig. 2 Angular modifying factor A_0 and multiplicative modifying factor K_0 as functions of cone half-angle for various freestream Mach numbers.

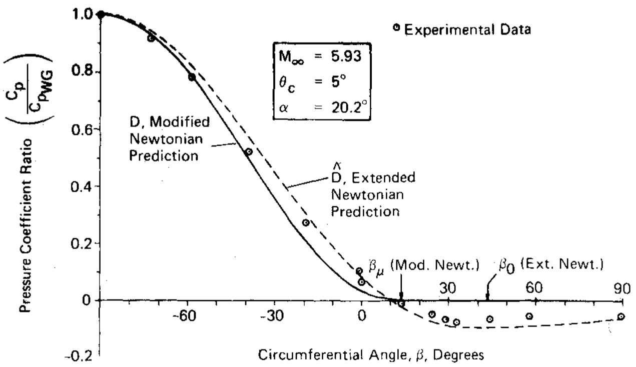

Fig. 3 Comparison of predicted pressure coefficient ratios with experimental data.

K_0, a multiplicative modifying factor, which is the ratio of the stagnation pressure coefficients through oblique and normal shock waves; and A_0, an angular modifying factor, which is the ratio of $\pi/2$ to the value of $(\alpha + \theta_c)$ which produces the maximum $C_{p_{WG}}$. Thus

$$\hat{C}_{p_{WG}} = K_0 C_{p_{STAG}} \sin^2[A_0(\alpha + \theta_c)] \quad (2)$$

Equations for K_0 and A_0 are derived in the paper, and their values for perfect gas flowfields are given in Fig. 2. The ratio of the $\hat{C}_p$ at any circumferential angle β to the value $\hat{C}_{p_{WG}}$ for the same flow conditions can be determined from Modified Newtonian Theory. These values are applicable on the windward portion of a cone for values of β up to the shadow angle, defined as $\beta_u = \sin^{-1}(\tan\theta_c/\tan\alpha)$. It is postulated that the steady-state pressure coefficient ratio can be expressed around the entire circumference (including the flow-separated region) by an equation of the form

$$\hat{D} = \hat{C}_p/\hat{C}_{p_{WG}} = D_1 - D_2 \sin\beta + D_3 \sin^2\beta \quad (3)$$

where $\hat{D} \equiv 1$ when $\beta = -\pi/2$. Defining β_o as the circumferential position of the minimum pressure coefficient ratio point, where $\partial\hat{D}/\partial\beta = 0$, and C_0 as the value of $\hat{D}$ at that point, then we express

$$D_1 = [\sin^2\beta_0 + C_0(1 + 2\sin\beta_0)]/\kappa$$
$$D_2 = 2\phi/\kappa \qquad D_3 = (1 - C_0)/\kappa \quad (4)$$
$$\phi \equiv (1 - C_0)\sin\beta_0 \qquad \kappa \equiv (1 + \sin\beta_0)^2$$

Finally, for this Extended Newtonian Theory, $\hat{C}_p \equiv \hat{C}_{p_{WG}}\hat{D}$.

In order to obtain the dynamic derivatives, the equations were rederived incorporating the effects of body motion and ignoring only the higher-order terms in the dynamic stability parameter. Relationships obtained during the steady-state analysis were applied, and the following complete expressions were obtained

$$(\hat{C}_{p_{WG}}) = K_0 K \sin^2[A_0(\alpha + \theta_c)] \times \left\{1 + \rho V_\infty\left[\frac{z_{CG}\sin\theta_c - x_{CG}\cos\theta_c + x\sec\theta_c}{\sin A_0(\alpha + \theta_c)}\right]\right\} \quad (5)$$

$$\hat{D} = D_1(1 + \psi) + D_2\left(1 + \frac{1-\phi}{2}\psi\right) + D_3(1 - \phi\psi) \quad (6)$$

where

$$\psi \equiv \rho V_\infty\left[\frac{z_{CG}\sin\alpha + x_{CG}\cos\alpha - x\sec^2\theta_c\cos\alpha}{\sin\alpha\cos\alpha(1 + \sin\beta_0)}\right]$$

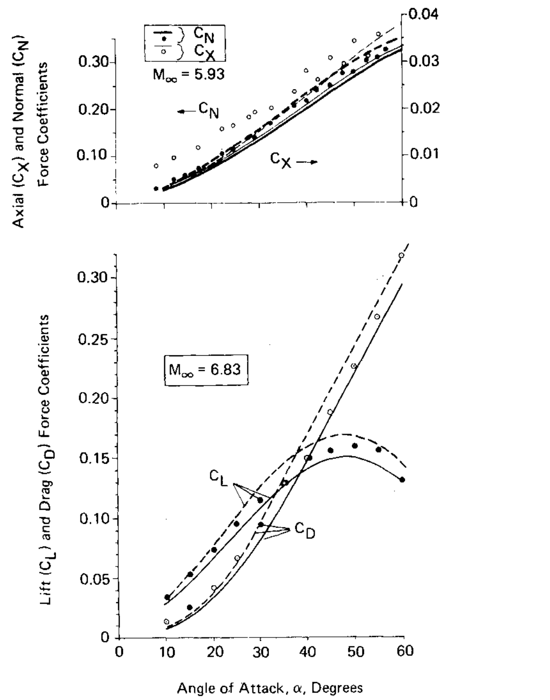

Fig. 4 Comparison of the predictions of Modified Newtonian (solid curves) and Extended Newtonian (dashed curves) Theories with experimental results; $\theta_c = 5$.

The complete expression by this Extended Newtonian Theory is therefore provided by Eqs. (4–6). The aerodynamic coefficients and the static and dynamic stability derivatives are now obtainable by appropriate integrations and differentiations.

The derived equations were applied to the case $\theta_c = 5°$ for freestream Mach numbers M_∞ of 5.93 and 6.83. For each M_∞, Eq. (6) was fitted to available circumferential pressure data by a least-squares technique, and C_0 was determined for a range of α, for those circumferential locations that minimized the resultant sum of the residuals. For $M_\infty = 5.93$, the following empirical expressions were obtained (see Fig. 3)

$$C_0 = 0.000423\alpha - 0.096 \qquad \beta_0 = -0.30\alpha + 44.0$$

where α is in degrees; for this case, $K_0 = 1.00011$, and $A_0 = 1.0207$. (Similar relations were obtained for $M_\infty = 6.83$.)

This approach yields static coefficients that agree more closely with the published experimental results than do the Modified Newtonian predictions through the α range from 10°–60° (Fig. 4), since it incorporates the effects of the flow-separated region. Further, dynamic stability derivatives may be obtained and related to Newtonian Impact Theory. The comparisons of the resulting aerodynamic characteristics (Fig. 4) reveals that the separated flow region has a significant effect upon slender hypersonic cones at high angles of attack.

Reference

[1] Amick, J. L., "Pressure Measurements on Sharp and Blunt 5°- and 15°-Half-Angle Cones at Mach Number 3.86 and Angles of Attack to 100°," TN D-753, Feb. 1961, NASA.